

Teaching Math with Meaning

Children with a diagnosis of autism vary greatly in their strengths/weaknesses, likes/dislikes and general personality and approach to life. There is no one teaching approach that will be successful with all children who fall into this diagnostic category. However, there are a variety of conventional math programs and ideas out there that can be modified according to an individual child's profile of skills and interests, to help that child learn math concepts with meaning.

The information in this handout is meant to give you ideas and starting points. I modify academic materials for many different ASD children - I have yet to use the same idea in exactly the same way for any two of them. As you design materials and activities for your child, be prepared to use "trial and error" to find out what approach is best. Don't be afraid of making mistakes - the result won't be fatal, and you can learn as much from what doesn't work as you do from what is successful. You need to step back and analyze why one activity made sense to your child, while another one did not, then modify and try again.

MATH IS A VISUAL AND MANIPULATIVE SKILL THAT IS ONLY IMPERFECTLY REPRESENTED BY LANGUAGE*

**** please note that some of the most difficult aspects of math for ASD students are "language of math" (words describing mathematical concepts), word problems (accurately translating from language to mathematical problem), estimation, and prediction***

In the school system, math skills are traditionally taught through language. For a child with weak language skills, the verbal explanations of math concepts are difficult to understand. In general, you will be most successful in teaching math concepts if you use concrete manipulatives and visual structures and demonstrations to allow the child to "see" the math concept. Connect visual concepts to numbers and number equations (eg. $2+3=5$), then add on the language. Always connect math activities to what they represent in the "real world" (eg. subtraction connects to spending money or using up the items in your fridge).

In the province of Ontario's new curriculum, there are 5 math "strands":

1. ***Number Sense and Numeration:*** counting, number recognition, more/less, part/whole, arithmetic operations (addition, subtraction facts to 20), place value (10's, 1's), half/whole, money, mental math (1-digit numbers), word problems, use of calculator, estimation
2. ***Measurement:*** units of measurement (time, weight, length, perimeter, area, volume, temperature), money units (eg. penny, nickel, dime), days of the week, months of the year, measurement comparisons, sequencing, estimation
3. ***Geometry and Spatial Sense:*** 2 & 3-dimensional shapes, symmetry, spatial concept words (eg. beside, under, left/right)
4. ***Patterning and Algebra:*** patterns (make, extend, describe), visual patterns, sound patterns, number patterns, pattern rules, translate pattern from one medium to another (eg. visual to sound), counting patterns (eg. on 100's chart)
5. ***Data Management and Probability:*** sort and classify items, collect and record data, charts, graphs (concrete graphs, pictographs), ask/answer questions based on charts/graphs, simple probability, prediction, surveys (yes/no questions)

1. Number Sense and Numeration

Counting with 1-to-1 correspondence

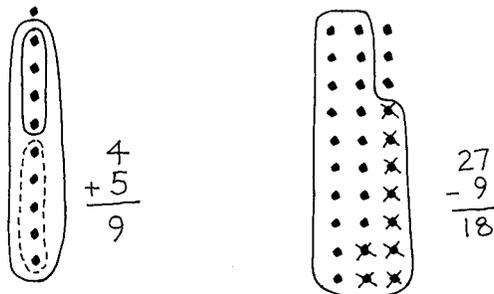
- introduce this in play situations
- use your child's interests to make sure you have 150% attention to the concept; eg. use a marble or car track, "give me (#)", "I want (#)"
- use flash cards as "visual jigs" (the card should have the numeral and then the number of spots that matches the numeral) - the child can use this to "check" if they've counted correctly
- gather bins of small interesting objects to be counted - what you choose depends on your child's interests (cars, beads, shiny "jewel" stones, shells, Dragon BallZ characters, etc.)
- count on a numberline - you can make it big and hop along it, you can put it on the wall and draw along it, you can add stickers to each number counted (to help the child pay attention to each number along the way)
- introduce the "language of math" - compare groups of objects - which are the SAME? which group has MORE? LESS? (please note that your child may cognitively understand the group comparisons, but may be unable to label them - or your child may use the words, but not flexibly understand the concepts they represent)

Addition/Subtraction

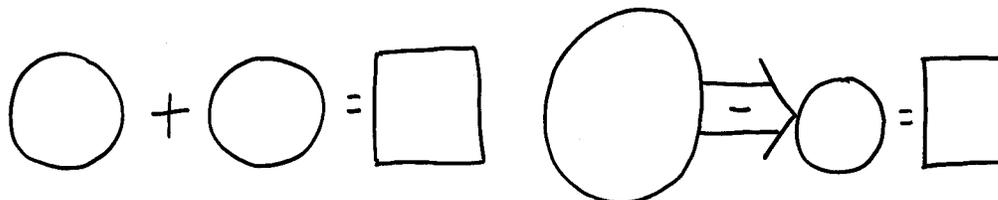
- always teach these concepts together
- work in "fact families" (eg. $1+4=5$ and $5-4=1$)
- "Math Their Way" is a manipulatives-based math program that teaches number concepts in a concrete visual way - this program exists in both school boards, and you may find the entire program, or portions of it, used in your child's classroom - the advantage of this program is that it is designed to teach groups of children, and if it is used in your child's school, you can adapt it with minimal effort (you will find few programs that you can use exactly as written - the modifications that you make will mostly have to do with changing any part of the program that is too "language-heavy") - the disadvantage of this program is that if it is not already set up, you will have to gather and make a certain number of materials to carry out the program completely - I've found that you can also use fragments of this program to reinforce certain concepts (eg. the part of the program that gets children to work with one number and find all the "group combinations" that make up that number - eg. "5" is "0 and 5", "1 and 4", "2 and 3", "3 and 2", "4 and 1" and "5 and 0" - once this is well understood, it is easier to teach addition/subtraction fact families with meaning)
- a math program that is being used locally in home schooling is called "Math-U-See" - you can find more information on this program on the internet at www.mathusee.com - this program uses math manipulatives to teach concepts from preschool to high school
- another program being used locally in the school system is called "Touch Math" - this program uses "touch points" on numerals to help children connect quantity and counting with the written number - once numbers are known, the program progresses to teaching addition, subtraction, multiplication and division - this can be an effective multi-sensory way to learn math facts - some children may find the "multi-mode" teaching style over-whelming, so you may not want to use the program exactly as written (there is a lot of auditory and language reinforcement that will be

helpful to some special learners, but irritating and over-whelming to others) - for effective connected math learning, you will have to do "extension teaching" to connect all math fact learning to what it represents in the "real world" (must always ensure that math forms are not taught without practical connection to math concepts - think of the number skills of the main character in the movie "Rain Man") - use manipulatives-based activities to check comprehension

- the "Child-Centered Math" books by Mary J. Kurth (Creative Teaching Press) also have many good ideas for visual experience-based teaching of early math concepts (again, you'll have to pick and choose amongst the activities, since some are more language-based than others)
- Another good book is "Math Wise" by Jim Overholt (The Center for Applied Research in Education) - this book contains ideas for teaching math concepts from kindergarten to grade 8 - many activities are suitable, some with modification - some activities are more language-based and abstract
- "unifix cubes" are a good investment (I wouldn't buy the cheaper "knock-off" versions of the cubes, because they don't fit together as well, and so are more difficult for the child to use) - these cubes can be used as manipulatives for counting, adding and subtracting - you can also use different colours of the cubes to teach place value (1's, 10's, 100's, etc)
- numberlines and "hundreds" charts can also be used to count forward to add to a number and backward to subtract from a number - you should be able to find these visual math supports in every primary classroom (I've also seen books of activities to use with the "hundreds" chart - you can ask at your school)
- dot matrices (single dots, rows of 10, and 10x10 dot arrays) are another good visual support to use for addition and subtraction (reference the "Math Wise" book for details of specific activities) - see below for examples:



- a white board and odourless dry erase markers are also useful support materials to have in the classroom - you can easily make simple "visual jigs" for use with concrete manipulatives - see below:



- for subtraction, you need to be able to "see" the items that are taken away, so that you can double-check the numbers in the equation

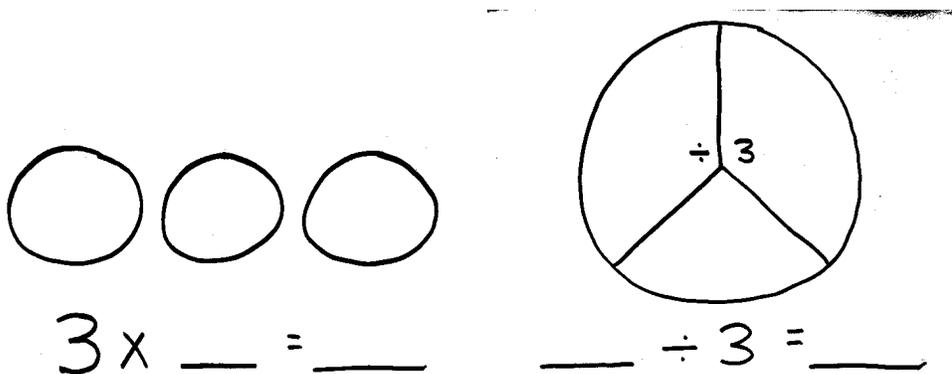
- try a "take-away cup" (drop subtracted items into the cup), also crossing-out pictures or dots in a dot matrix or numbers in a number line will work for some children (this method will not make sense to others who rightly decide that the items are still there, they just have extra marks on them) - you can also try items/shapes molded from plasticene or playdoh, where the subtracted items are "squished"

"Skip" Counting

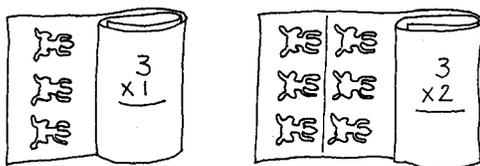
- the "base" of understanding multiplication and division is counting by 2's, 3's, 4's, 5's, etc, so all games involving these skills will help prepare for the teaching of these later math skills

Multiplication/Division

- as with addition and subtraction, teach these skills together
- the "base" of understanding multiplication is counting by 2's, 3's, 4's, 5's, etc, so all games involving these skills will help prepare for the teaching of multiplication
- multiplication is repeated addition of the same number
- division is repeated subtraction of the same number
- help the child to "picture" what is being done with the groups of objects by using "visual jigs" and concrete manipulatives - see examples below:



- use real objects, stickers, unifix cubes with these "jigs" - have child make an equation to match the picture, or a picture to match the equation
- you can also use a numberline or "hundreds" chart with a highlighter to mark the repeated addition (multiplication) or subtraction (division) that the equation represents
- another good visual for multiplication is a "fold-out" fact family (idea from "Math Wise") - see below:



Word Problems

- word problems are difficult for ASD children, because of their language weaknesses
- your job is to help your child "translate" the language into a picture of the math question that is being asked
- you can "play out" the math question using small toys (some ideas for this are in "Child-Centered Math" books by Mary J. Kurth) - for example, using a road-map rug and small cars, you can play out "2 cars were at the store. 4 more cars drive into the store parking lot. How many cars in all?" - write the equation on a white-board beside your play area
- older children can draw, use stickers, or "cut & paste" to represent the problem
- you need to go from language to picture to equation
- I would also recommend starting a "Math Journal" where you define (in words and pictures and numbers) common "math terms" - eg. "how many IN ALL?", "how many ARE LEFT?", MORE/LESS, FIRST/SECOND/LAST

Place Value

- basic knowledge built by understanding manipulatives in groups of 10 (eg. straws representing number of days in school)
- unifix cubes - can colour code for place value (practice making numbers with the blocks)
- dot matrices (see "Math Wise")
- teach equivalency: $10 = 10 \times (1)$; $100 = 10 \times (10)$; $1000 = 10 \times (100)$ - practice these concepts using manipulatives
- need to learn to read and "break down" larger numbers (practice representing 1, 2 and 3-digit numbers using unifix cubes and dot matrices)
- "carry-over" in addition and subtraction can be a difficult concept to teach - use a tangible exchange (eg. "carry-over cup" where a blue (10) cube can be exchanged for 10 red (1) cubes)

Fractions

- divide and cut real objects (eg. paper, playdoh, food)
- comprehension of fractions is tied to understanding of division

2. Measurement

- do "real-world" experiments and record
- may want to start by measuring the child's own body, and recording measurements on a large chart (allows some sort of comparison, eg. "that object is longer/shorter than my arm")
- compare measurements of various objects and connect concepts to math language (eg. bigger/smaller, longer/shorter, heavier/lighter)
- generally ASD children have a poor sense of time (eg. don't have an intuitive sense of how long a minute, 5 minutes, an hour is)
- connect teaching of time to measuring and tracking events that are significant to the child (eg. daily schedule, calendar of personal events, timing how long it takes for the oil to drop through a colourful visual timer)
- relate all measurement to child's own experience (eg. temperature - connect to seasons, weather, clothing, activities, etc.)

3. Geometry and Spatial Sense

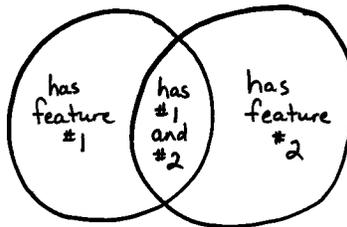
- find shapes in everyday objects and pictures (can go on a "shape hunt" in your neighbourhood, or in the halls of the school - give the child a visual picture or cut-out of the shape you are looking for - record what you find) - match visual shapes to words for those shapes
- sort 2-D and 3-D shapes according to specific features (eg. 4 sides, square corners)
- play games with 2-D and 3-D shapes (eg. tangrams, blocks, flat magnet shapes that can be used to make pictures)
- use shapes to make structures (can use building toys like K'nex, Lego)
- teach spatial concept words (eg. below, on top, beside) - can use pictures and real objects to allow child to demonstrate knowledge - you can have the child move themselves in space (eg. on the playground climber)
- symmetry - use mirror experiments, draw, complete a whole picture when given half - find symmetrical objects and shapes in the child's environment
- transformations - draw, move shapes in space (eg. to demonstrate the difference between a "flip" and a "slide")
- mapping (grids, coordinates) - place objects on a map according to the number coordinates, write coordinates for objects on the map - connect to maps of real places (teach child how to interpret a map of the back-yard, the neighbourhood, the city) - you can make map-making and map-reading fun by burying "treasure" and then following the map to find it (connect the 2-D paper map to the 3-D place that it represents)

4. Patterning and Algebra

- do patterning activities across sensory modalities (visual, auditory, tactile)
- copy and extend the patterns of others - make up own patterns
- translate patterns from one sensory modality to another (eg. change a visual pattern into a matching auditory pattern)
- child may have difficulty with the language used to describe pattern rules
- counting patterns can be explored using the "hundreds" chart and numberline (eg. counting by 2's, 3's, 4's - use a highlighter to mark the numbers that fall into the pattern)
- number patterns can be explored using the "hundreds" chart and numberline (eg. the pattern rule might be "+2, starting at 8" - use a highlighter to mark the numbers that fall into the pattern)
- can do calculation tables and explore the patterns in them (see "Mathquest" textbook - I believe this type of activity starts around grade 3)
- understanding number patterns, and how the rules are expressed with numbers and symbols, is a necessary underlying skill for later comprehension of algebra equations

5. Data Management and Probability

- earliest skill in this area is the ability to sort and classify items into various categories according to specific features of the item (eg. colour, size, function, etc)
- gather lots of items and have the child sort them into labelled boxes, or circles on a page - start with only 2 categories (eg. "red" and "not red")
- sort the same group of items according to different features (helps child to see that the result is flexible, depending on what is the critical sorting feature - eg. you'll get different groups when you sort for colour than you do when you sort by size)
- Venn diagrams provide a visual "jig" for sorting by 2 features at the same time - see below:



- graphs are visual representation of "real world" data
- from the beginning, make the connection between the graph and what it represents (objects or pictured objects)
- activities should go both ways - when given the concrete/pictured items, the child makes a graph - when given a graph, the child can choose the correct number of real or pictured items to match the information in that graph
- you can use drawing, stickers or "cut & paste" for graphing activities
- the most difficult part of graphing for ASD children is answering questions based on the graph (again, we get into the "language of math")
- probability is also a difficult concept for most children in the autism spectrum, since it requires guessing and estimating (you may see some distress when you try to introduce these concepts - try activities that are "investigation-based", eg. rolling a dice or flipping a coin and recording the results)

Extra Resources:

Check the internet for sites where teachers (and home-schooling parents) exchange ideas for lessons on a variety of subjects - here are two that I found in my browsing:

1. <http://teachers.net/lessons/> - *this is a site where teachers exchange lesson ideas on many subjects (activities are organized by grade level)*
2. <http://homeschoolcentral.com/special.htm> - *this is a page that lists many connections for parents who are home-schooling children with special needs*